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Short Communication

ON FUZZY COMPLEX DERIVATIVES II

Pishtiwan O. Sabir
Department of Mathematics, Faculty of Science and Science Education
University of Sulaimani, Sulaimani, Kurdistan Region, Iraq

ABSTRACT

In this paper, an important theorem of fuzzy derivative for fuzzy complex functions which map a regular complex numbers into bounded closed complex complement normalized fuzzy numbers is proved. This is a modification and generalization of the fuzzy derivative in Sabir *et al.* (2012).

Keywords: Fuzzy complex numbers, Fuzzy Functions, Fuzzy derivatives.

INTRODUCTION

Fuzzy complex analysis was first introduced by Buckley and Qu (1991, 1992) that extends definitions and results of Dubois and Prade (1982) to the complex case. Buckley (1989) suggested the notion of convergence, differentiation and continuity of complex fuzzy function (Guangquan, 1991; Chun and Ma, 1998; Dianjun, 2000; Qiu *et al.*, 2009; Ousmane and Congxin, 2003; Shengquan, 2006; Cai, 2009). As a generalization of Buckley's work, several scholars continued research in fuzzy analysis like Wu and Qiu (1999), Zengtai and Shengquan (2006), Qiu and Shu (2008), Sun and Guo (2010), Ma and Chen (2012) and Sabir (2012).

PRILIMINARIES

Zadeh (1965) firstly introduced the concept of fuzzy subset which is a function $\mu(\tilde{A}, x): X \to [0,1]$ and a generalization of the classical set operations.

Definition 1. Let $\tilde{A}$ be a fuzzy subset and $\alpha \in [0,1]$, then
(1) The α–level of $\tilde{A}$, denoted by ${}^{\alpha+}\tilde{A}$, is the crisp set $\{x \in X: \mu(\tilde{A}, x) \geq \alpha\}$.
(2) The weak α–level ${}^{\alpha-}\tilde{A}$ of a fuzzy subset $\tilde{A}$ is the non-fuzzy set of all elements of X that grade of memberships are greater than α.
(3) The height of a fuzzy subset $\tilde{A}$ is the number obtained by $\alpha_{\tilde{A}}^{max} = sup_{x \in X}\, \mu(\tilde{A}, x)$.

Definition 2. For any collection, $\{\tilde{A}_i : i \in I\}$, of fuzzy subsets of X, where I is a nonempty index set.
(1) The union of fuzzy subsets $\tilde{A}_i$ is defined by $\mu(\bigcup_i \tilde{A}_i, x) = sup_x\, \mu(\tilde{A}_i, x)$.
(2) The intersection of fuzzy subsets $\tilde{A}_i$ is defined by

$$\mu(\bigcap_i \tilde{A}_i, x) = inf_x\, \mu(\tilde{A}_i, x).$$

(3) The complement of $\tilde{A}_i$ is defined by $\mu(\neg\tilde{A}_i, x) = 1 - \mu(\tilde{A}_i, x)$, for all x belongs to X.

Definition 3. A fuzzy number $\tilde{a}$ defined on the set of real numbers $\mathcal{R}$ is a function $\mu(\tilde{a}, x): \mathcal{R} \to [0,1]$, which satisfies:
(1) $\tilde{a}$ is upper semicontinuous.
(2) $\mu(\tilde{a}, x) = 0$ outside some interval $[c, d]$.
(3) There are real numbers a, b such that $c \leq a \leq b \leq d$, $\mu(\tilde{a}, x)$ is increasing on $[\mathrm{c}, \mathrm{a}]$, $\mu(\tilde{a}, x)$ is decreasing on $[b, d]$, and $\mu(\tilde{a}, x) = 1$, $a \leq x \leq b$.

Definition 4. A fuzzy complex number $\hat{Z}$ is a mapping $\mu(\hat{Z}, z): \mathbb{C} \to [0,1]$ if and only if:
(1) $\mu(\hat{Z}, z)$ is continuous.
(2) ${}^{\alpha-}\hat{Z}$ is open, bounded, and connected.
(3) ${}^{1+}\hat{Z}$ is non-empty, compact, and arcwise connected.

RESULTS

Definitions, results and notations on fuzzy complex analysis which are used in this section can be found in Sabir *et al.* (2012).

Theorem 1. Let $\tilde{Z}, \widetilde{W}$ be BCCCNFNs and $\tilde{f}$ be a fuzzy meromorphic function. If $\tilde{f}(z) = \widetilde{W}$ and $\tilde{f}'(z) = \widetilde{W}$ have the same zeros, $\tilde{f}(z) = \tilde{Z}$ and $\tilde{f}'(z) = \tilde{Z}$ have the same zeros with the same order, and $\overline{N}(r, {}^{z,\gamma+}\tilde{f}) = o(T(r, {}^{z,\gamma+}\tilde{f}))$ then ${}^{z,\gamma+}\dot{\tilde{f}} = {}^{z,\gamma+}\tilde{f}$.

Proof: By hypothesis, we have

$$2T(r, {}^{z,\gamma+}\tilde{f}) \leq \overline{N}(r, {}^{z,\gamma+}\tilde{f}) + \overline{N}\left(r, \frac{1}{{}^{z,\gamma+}\tilde{f} - {}^{\gamma+}\tilde{Z}}\right)$$

*Corresponding author email: pishtiwan.sabir@gmail.com

$$+\overline{N}\left(r,\frac{1}{{}^{z,\gamma+}\tilde{f}-{}^{\gamma+}\widetilde{W}}\right)+\overline{N}\left(r,\frac{1}{{}^{z,\gamma+}\tilde{f}}\right)+o\big(T(r,{}^{z,\gamma+}\tilde{f})\big)$$

$$=\overline{N}\left(r,\frac{1}{{}^{z,\gamma+}\tilde{f}}\right)+\overline{N}\left(r,\frac{1}{{}^{z,\gamma+}\dot{\tilde{f}}-{}^{\gamma+}\tilde{Z}}\right)+\overline{N}\left(r,\frac{1}{{}^{z,\gamma+}\dot{\tilde{f}}-{}^{\gamma+}\widetilde{W}}\right)+o\big(T(r,{}^{z,\gamma+}\tilde{f})\big)$$

$$=\overline{N}\left(r,\frac{1}{{}^{z,\gamma+}\tilde{f}}\right)+N\left(r,\frac{{}^{z,\gamma+}\dot{\tilde{f}}}{{}^{z,\gamma+}\tilde{f}}\right)+o\big(T(r,{}^{z,\gamma+}\tilde{f})\big)$$

$$=\overline{N}\left(r,\frac{1}{{}^{z,\gamma+}\tilde{f}}\right)+\overline{N}\big(r,{}^{z,\gamma+}\tilde{f}\big)+\overline{N}\left(r,\frac{1}{{}^{z,\gamma+}\tilde{f}}\right)+o\big(T(r,{}^{z,\gamma+}\tilde{f})\big)$$

$$=2\overline{N}\left(r,\frac{1}{{}^{z,\gamma+}\tilde{f}}\right)+o\big(T(r,{}^{z,\gamma+}\tilde{f})\big)$$

$$\leq 2T\big(r,{}^{z,\gamma+}\tilde{f}\big)+o\big(T(r,{}^{z,\gamma+}\tilde{f})\big).$$

It follows that,

$$\overline{N}\left(r,\frac{1}{{}^{z,\gamma+}\tilde{f}}\right)=\overline{N}\left(r,\frac{1}{{}^{z,\gamma+}\dot{\tilde{f}}-{}^{\gamma+}\tilde{Z}}\right)+\overline{N}\left(r,\frac{1}{{}^{z,\gamma+}\dot{\tilde{f}}-{}^{\gamma+}\widetilde{W}}\right)+o\big(T(r,{}^{z,\gamma+}\tilde{f})\big),$$ and

$$m\left(r,\frac{1}{{}^{z,\gamma+}\tilde{f}-{}^{\gamma+}\widetilde{W}}\right)$$

$$\leq m\left(r,\frac{\big({}^{z,\gamma+}\dot{\tilde{f}}-{}^{\gamma+}\tilde{Z}\big)-\big({}^{z,\gamma+}\tilde{f}-{}^{\gamma+}\tilde{Z}\big)}{{}^{z,\gamma+}\dot{\tilde{f}}\big({}^{z,\gamma+}\dot{\tilde{f}}-{}^{\gamma+}\tilde{Z}\big)-{}^{z,\gamma+}\ddot{\tilde{f}}\big({}^{z,\gamma+}\tilde{f}-{}^{\gamma+}\tilde{Z}\big)}\right)$$

$$+m\left(r,\frac{{}^{z,\gamma+}\dot{\tilde{f}}}{\big({}^{z,\gamma+}\tilde{f}-{}^{\gamma+}\tilde{Z}\big)\big({}^{z,\gamma+}\tilde{f}-{}^{\gamma+}\widetilde{W}\big)}\right)$$

$$+m\left(r,\frac{{}^{z,\gamma+}\ddot{\tilde{f}}}{{}^{z,\gamma+}\dot{\tilde{f}}\big({}^{z,\gamma+}\dot{\tilde{f}}-{}^{\gamma+}\tilde{Z}\big)}\right)+m\left(r,\frac{{}^{z,\gamma+}\dot{\tilde{f}}}{\big({}^{z,\gamma+}\tilde{f}-{}^{\gamma+}\widetilde{W}\big)}\right)$$

$$\leq T\left(r,\frac{{}^{z,\gamma+}\dot{\tilde{f}}}{{}^{z,\gamma+}\tilde{f}-{}^{\gamma+}\tilde{Z}}-\frac{{}^{z,\gamma+}\ddot{\tilde{f}}}{{}^{z,\gamma+}\dot{\tilde{f}}-{}^{\gamma+}\tilde{Z}}\right)+o\left(T\big(r,{}^{z,\gamma+}\tilde{f}\big)\right)$$

$$+o\big(T(r,{}^{z,\gamma+}\tilde{f})\big)+o\left(T(r,{}^{z,\gamma+}\dot{\tilde{f}})\right)$$

$$=N\left(r,\frac{{}^{z,\gamma+}\dot{\tilde{f}}}{{}^{z,\gamma+}\tilde{f}-{}^{\gamma+}\tilde{Z}}-\frac{{}^{z,\gamma+}\ddot{\tilde{f}}}{{}^{z,\gamma+}\dot{\tilde{f}}-{}^{\gamma+}\tilde{Z}}\right)$$

$$+m\left(r,\frac{{}^{z,\gamma+}\dot{\tilde{f}}}{{}^{z,\gamma+}\tilde{f}-{}^{\gamma+}\tilde{Z}}-\frac{{}^{z,\gamma+}\ddot{\tilde{f}}}{{}^{z,\gamma+}\dot{\tilde{f}}-{}^{\gamma+}\tilde{Z}}\right)$$

$$+o\big(T(r,{}^{z,\gamma+}\tilde{f})\big)$$

$$\leq \overline{N}\big(r,{}^{z,\gamma+}\tilde{f}\big)+m\left(r,\frac{{}^{z,\gamma+}\dot{\tilde{f}}}{{}^{z,\gamma+}\tilde{f}-{}^{\gamma+}\tilde{Z}}\right)+m\left(r,\frac{{}^{z,\gamma+}\ddot{\tilde{f}}}{{}^{z,\gamma+}\dot{\tilde{f}}-{}^{\gamma+}\tilde{Z}}\right)$$

$$+o\big(T(r,{}^{z,\gamma+}\tilde{f})\big)$$

$$=o\left(T\big(r,{}^{z,\gamma+}\tilde{f}\big)\right)+o\left(T\big(r,{}^{z,\gamma+}\tilde{f}\big)\right)+o\left(T\left(r,{}^{z,\gamma+}\dot{\tilde{f}}\right)\right)$$

$$+o\big(T(r,{}^{z,\gamma+}\tilde{f})\big).$$

Hence,

$$N\left(r,\frac{1}{{}^{z,\gamma+}\dot{\tilde{f}}-{}^{\gamma+}\tilde{Z}}\right)+N\left(r,\frac{1}{{}^{z,\gamma+}\tilde{f}-{}^{\gamma+}\widetilde{W}}\right)+o\left(T\big(r,{}^{z,\gamma+}\tilde{f}\big)\right)$$

$$=N\left(r,\frac{1}{{}^{z,\gamma+}\tilde{f}-{}^{\gamma+}\tilde{Z}}\right)+N\left(r,\frac{1}{{}^{z,\gamma+}\tilde{f}-{}^{\gamma+}\widetilde{W}}\right)+o\left(T\big(r,{}^{z,\gamma+}\tilde{f}\big)\right)$$

$$=N\left(r,\frac{1}{{}^{z,\gamma+}\tilde{f}-{}^{\gamma+}\widetilde{W}}\right)+o\left(T\big(r,{}^{z,\gamma+}\tilde{f}\big)\right),$$ and

$$m\left(r,\frac{1}{{}^{z,\gamma+}\dot{\tilde{f}}-{}^{\gamma+}\widetilde{W}}\right)+N\left(r,\frac{1}{{}^{z,\gamma+}\dot{\tilde{f}}-{}^{\gamma+}\widetilde{W}}\right)$$

$$\leq T\big(r,{}^{z,\gamma+}\tilde{f}\big)+o\left(T\big(r,{}^{z,\gamma+}\tilde{f}\big)\right)$$

$$=\overline{N}\left(r,\frac{1}{{}^{z,\gamma+}\tilde{f}-{}^{\gamma+}\widetilde{W}}\right)+o\left(T\big(r,{}^{z,\gamma+}\tilde{f}\big)\right)$$

$$\leq N\left(r,\frac{1}{{}^{z,\gamma+}\dot{\tilde{f}}-{}^{\gamma+}\widetilde{W}}\right)+o\left(T\big(r,{}^{z,\gamma+}\tilde{f}\big)\right).$$

Therefore,

$$T\left(r,\frac{1}{{}^{z,\gamma+}\tilde{f}-{}^{\gamma+}\tilde{Z}}\right)+T\left(r,\frac{1}{{}^{z,\gamma+}\dot{\tilde{f}}-{}^{\gamma+}\tilde{Z}}\right)+T\left(r,\frac{1}{{}^{z,\gamma+}\tilde{f}-{}^{\gamma+}\widetilde{W}}\right)$$

$$+T\left(r,\frac{1}{{}^{z,\gamma+}\dot{\tilde{f}}-{}^{\gamma+}\widetilde{W}}\right)$$

$$=m\left(r,\frac{1}{{}^{z,\gamma+}\tilde{f}-{}^{\gamma+}\tilde{Z}}\right)+m\left(r,\frac{1}{{}^{z,\gamma+}\dot{\tilde{f}}-{}^{\gamma+}\tilde{Z}}\right)$$

$$+m\left(r,\frac{1}{{}^{z,\gamma+}\tilde{f}-{}^{\gamma+}\widetilde{W}}\right)+m\left(r,\frac{1}{{}^{z,\gamma+}\dot{\tilde{f}}-{}^{\gamma+}\widetilde{W}}\right)$$

$$+N\left(r,\frac{1}{{}^{z,\gamma+}\tilde{f}-{}^{\gamma+}\tilde{Z}}\right)+N\left(r,\frac{1}{{}^{z,\gamma+}\dot{\tilde{f}}-{}^{\gamma+}\tilde{Z}}\right)$$

$$+N\left(r,\frac{1}{{}^{z,\gamma+}\tilde{f}-{}^{\gamma+}\widetilde{W}}\right)+N\left(r,\frac{1}{{}^{z,\gamma+}\dot{\tilde{f}}-{}^{\gamma+}\widetilde{W}}\right)$$

$$+o\left(T\big(r,{}^{z,\gamma+}\tilde{f}\big)\right)$$

$$\leq m\left(r,\frac{1}{{}^{z,\gamma+}\ddot{\tilde{f}}}\right)-m\left(r,\frac{1}{{}^{z,\gamma+}\dot{\tilde{f}}}\right)+N\left(r,\frac{1}{{}^{z,\gamma+}\dot{\tilde{f}}-{}^{\gamma+}\widetilde{W}}\right)$$

$$+m\left(r,\frac{1}{{}^{z,\gamma+}\dot{\tilde{f}}}\right)+T\big(r,{}^{z,\gamma+}\tilde{f}\big)+o\left(T\big(r,{}^{z,\gamma+}\tilde{f}\big)\right)$$

$$\leq T\left(r, {}^{z,\gamma+}\dot{\tilde{f}}\right) + T\left(r, \frac{1}{{}^{z,\gamma+}\dot{\tilde{f}} - {}^{\gamma+}\tilde{W}}\right) + T\left(r, {}^{z,\gamma+}\tilde{f}\right)$$

$$+o\left(T\left(r, {}^{z,\gamma+}\tilde{f}\right)\right) \text{ a Contradiction.}$$

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